MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2015/2016

PCM0235 - CALCULUS

(Foundation in Information Technology/ Foundation in Life Sciences)

8 MARCH 2016
2.30 p.m – 4.30 p.m
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This Question paper consists of 3 pages excluding the cover page and appendix.
2. Answer **FIVE** out of **SIX** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Write all your answers in the Answer Booklet provided.

Answer any five questions.

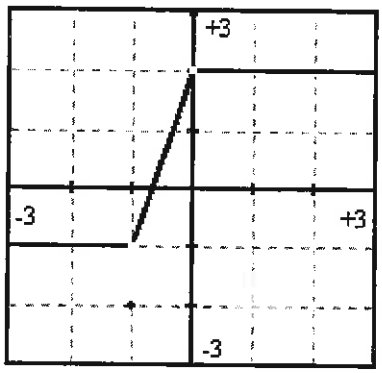
Question 1 (10 marks)

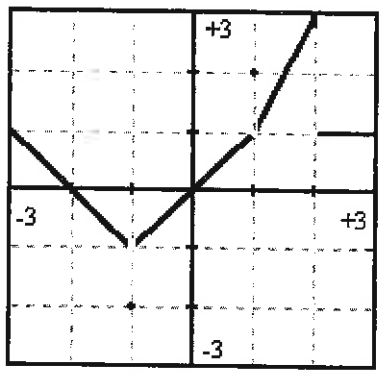
a) Evaluate the following limits.

i. $\lim_{x\to -1}\dfrac{(2x-1)^2-9}{x+1}$ (2 marks)

ii. $\lim_{\theta\to 0}\dfrac{\dfrac{1}{2+\sin\theta}-\dfrac{1}{2}}{\sin 2\theta}$ (4 marks)

b) Given the graph of two functions.

$f(x)$ 

and $g(x)$ 

Find

i. $\lim_{x\to -1} f(x)$ (1 mark)

ii. $\lim_{x\to 2} g(x)$ (1 mark)

iii $g(1)+\lim_{x\to 1}[f(x)g(x)]$ (2 marks)

Question 2 (10 marks)

a) Find $\dfrac{dy}{dx}$ for $y=2x^3\tan 2x$. (2 marks)

b) Use implicit differentiation to find $\dfrac{dy}{dx}$, given $y^2+2y=x^3-2x^2$. Hence find the equation of the tangent line at $x=0$. (5 marks)

c) Find $f''(x)$ for $f(x)=\ln(x^2+4x)$. (3 marks)

Continued...

Question 3 (10 marks)

Given $y = 4x^3 - 3x^4$

a) Find the interval on which the function is increasing and the interval on which the function is decreasing. Identify the x-coordinates of any local maximum points and local minimum points. (4 marks)

b) Find the interval on which the function is concave up and interval on which the function is concave down. (4 marks)

c) Sketch the graph of the function. Show the x-intercepts. (2 marks)

Question 4 (10 marks)

a) Find:

i. $\int_{-2}^{1} \left(x^2 + 2|x|\right) dx$. (4 marks)

ii. $\int \frac{2\ln(x+1)}{x+1} dx$. (2 marks)

b) Use partial fraction decomposition to evaluate $\int \frac{x^2 + x + 2}{3x^3 + 9x^2 - 12x} dx$. (4 marks)

Question 5 (10 marks)

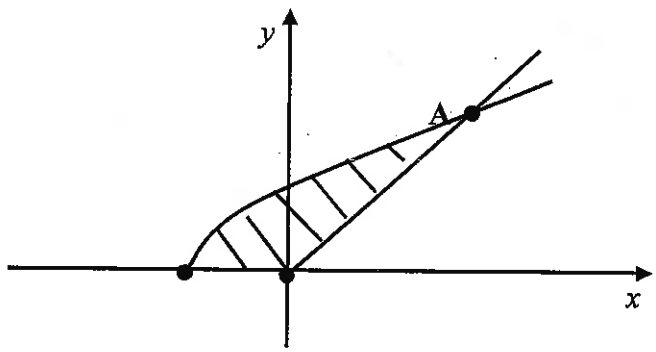
a) A region is bounded by $y = -(x^2 + 4), y = 0, x = -2$ and $x = 3$.

i. Sketch the graphs of the functions and identify the region. (2 marks)

ii. Find the area of the bounded region. (2 marks)

Continued...

b) A region bounded by $y = \sqrt{x+2}, y = x$ and x-axis is shown below.



i. Find point A. (1 mark)

ii. Find the volume of the solid of revolution when the region is rotated about the x–axis. (5 marks)

Question 6 (10 marks)

a) Solve the first order differential equation, $\frac{dy}{dx} = \frac{4x^2}{y + \cos y}$ given $y(0) = \pi$.

(4 marks)

b) Solve the second order differential equation, $y''-6y'+9y = 0$ given $y(0) = 4$ and $y'(0) = 2$. (6 marks)

End of Page

APPENDIX

A. Differentiation Rules

$\frac{d}{dx}[x^n] = nx^{n-1}$; n is any real number

$\frac{d}{dx}[f(x).g(x)] = f(x)g'(x) + f'(x)g(x)$; The Product Rule

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$; The Quotient Rule

$\frac{d}{dx}[f(g(x)] = f'(g(x)).g'(x)$; The Chain Rule

$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}.g'(x)$; The power rule combined with the chain rule:

$\frac{d}{dx}[\sin x] = \cos x$ $\quad$ $\frac{d}{dx}[\cos x] = -\sin x$ $\quad$ $\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$ $\quad$ $\frac{d}{dx}[\cot x] = -\csc^2 x$ $\quad$ $\frac{d}{dx}[\csc x] = -\csc x \cot x$

$\frac{d}{dx}[e^x] = e^x$ $\quad$ $\frac{d}{dx}[\ln x] = \frac{1}{x}; \quad x > 0$

B. Basic Integration Formulas

$\int cf(x)\,dx = c\int f(x)\,dx$ $\quad$ $\int k.dx = kx + C$

$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$ $\quad$ $\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$

$\int e^x\,dx = e^x + C$ $\quad$ $\int \frac{1}{x}dx = \ln|x| + C$

Integration by-parts: $\int u\,dv = uv - \int v\,du$

Volume (disk) $= \pi\int_a^b (f(x))^2\,dx$ $\quad$ **Area** $= \int_a^b (f(x) - g(x))dx$

Volume (washer) $= \pi\int_a^b \left[(f(x))^2 - (g(x))^2\right]dx$